

Revealing the black body regime of small x DIS through final state signals

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Abstract

We derive the major characteristics of inclusive and diffractive final states in Deep Inelastic Scattering off heavy nuclei for the high energy (small x) kinematics in which the limit of complete absorption is reached for the dominant hadronic fluctuations in the virtual photon (the black body limit of the process). Both the longitudinal and transverse distributions of the leading hadrons are found to be strikingly different from the corresponding ones within the leading twist approximation, and hence provide unambiguous signals for the onset of the black body limit.

The black body limit (BBL) of Deep Inelastic Scattering (DIS) from a heavy nucleus was first considered by V. Gribov in 1969 [1], before the discovery of perturbative QCD

(pQCD). He assumed that at ultra high energies each hadronic configuration in the photon scatters with the same strength, and is absorbed by the nucleus. If the (virtual) photon is expressed as a linear superposition of partonic components (characterised by the transverse and longitudinal momenta of the individual partons) then the interaction with the target does not mix up these partonic components, i.e. they are eigenstates of the scattering matrix. This implies that the interaction is diagonal for each hadronic fluctuation and features such as transverse sizes, invariant masses, and longitudinal momentum distributions are preserved by the BBL interaction.

The proton structure function, $F_2^p(x, Q^2)$, measured in DIS at HERA exhibits a rapid rise with the energy of the photon-proton subprocess, W , at high energies, or small x ($x = -q^2/2p \cdot q = Q^2/(W^2 + Q^2 - m_p^2)$, where p, m_p are the proton's momentum and mass, and $q, q^2 = -Q^2$ are the photon's momentum and virtuality). The standard leading twist approximation (LTA) of pQCD describes F_2^p extremely successfully in terms of a convolution of perturbatively calculable hard coefficient functions with quark and gluon parton distributions, which evolve in scale according to the DGLAP [2] evolution equations. Hence the rise of F_2^p is translated into a rapid rise of the gluon and sea-quark distributions of the proton with $1/x$ at small x . To leading-log accuracy in Q^2 , this leads to numerically large density of partons at small x , which calls into question the applicability of the LTA at even smaller x (for recent reviews see [3–6]). This concern is best understood in the target rest frame by considering the inelastic interaction cross sections for given partonic configurations of the virtual photon. In particular, the expression for the scattering of a small $q\bar{q}$ dipole, of transverse diameter d_\perp at small x is given, in terms of the leading-log gluon distribution of the nuclear target, xg^A , by:

$$\hat{\sigma}_{\text{pQCD}}^{\text{inel}}(d_\perp^2, x) = \frac{\pi^2}{3} d_\perp^2 \alpha_s(\bar{Q}^2) xg^A(x, \bar{Q}^2), \quad (1)$$

in which the scale $\bar{Q}^2 = \lambda/d_\perp^2$, with λ a logarithmic function of Q^2 . For large gluon densities, this appears to lead to a conflict with unitarity of the S-matrix for the interaction of spatially small wave packets in the virtual photon which are almost completely absorbed (especially

in the case of heavy nuclei, where the gluon density is enhanced by nucleon number A). This may be conveniently expressed in terms of the geometrical limit for the inelastic “small dipole” - nucleus interaction of

$$\hat{\sigma}_{\text{pQCD}}^{\text{inel}} \leq \hat{\sigma}_{\text{black}} = \pi R_{\text{target}}^2, \quad (2)$$

corresponding to the BBL for the total cross section of $2\pi R_{\text{target}}^2$.

Numerical studies performed using eqs. (1, 2) have demonstrated that interactions at the upper range of HERA energies in gluon-induced processes may be fairly close to the BBL, see e.g. [6,7]. This is consistent with an analysis [8] of current fits to HERA diffractive data, which lead to diffractive gluon densities which are significantly larger than the quark ones (see also [9]).

For heavy nuclei the perturbative x, Q^2 region, in which the interaction should be close to the BBL in both quark and gluon channels, is much broader than for nucleons. For example, for DIS on Pb at $x \sim 10^{-3}$, at small impact parameters the BBL may be reached for $Q^2 \leq 7(15)\text{GeV}^2$ for the quark (gluon) channel, that is for the region where $\ln Q^2/\Lambda_{\text{QCD}}^2 \geq \ln x_0/x$ ($x_0 \sim 0.05 \div 0.1$ is the starting scale of the evolution at small x). This indicates that DGLAP becomes inapplicable in an x, Q^2 range where expansion of a small dipole in the transverse direction due to gluon bremsstrahlung (diffusion) is still small and hence the coupling constant is small. This indicates an exciting possibility of the existence of the BBL in QCD in the perturbative domain for the kinematics which will be probed at LHC and which could also be probed at RHIC in pA collisions and at HERA/THERA in eA mode.

The experimental and theoretical challenge is to find unambiguous signals for the BBL. One problem is that predictions for the inclusive structure functions do not contain “smoking gun” signatures since they must account for leading twist shadowing, uncertainties in the input gluon densities, etc. This is especially acute in the case of the nucleon scattering where, even in the BBL, the structure function $F_2^p(x, Q^2)$ is expected to increase rather rapidly, $F_2^p \propto \ln^3 1/x$, of which $\ln^2 1/x$ is due to a Froissart-like increase of the radius of interaction $r_{\text{int}} \propto \ln 1/x$ and the remaining $\ln 1/x$ reflects the infinite renormalization of the

electric charge [1]. As discussed in [7], the latter results from the fact that the photon couples in a point-like manner to quarks. This also implies that for any energy there will always be small configurations in the photon wave function which have not yet reached the BBL. However, we expect that the BBL formulae will give a reasonable description of structure functions and hard diffractive processes for heavy nuclear target, where the blackness of the interaction is enhanced by the number of nucleons. The growth in the radius is essentially screened by the nuclear environment so that for large nuclei: $F_2^A \propto \ln 1/x$.

To deduce predictions for the properties of the final states it is useful to work within the $q\bar{q}$ dipole approximation for the photon wave function. In the BBL the elastic and inelastic contributions are equal so that the total cross section is given by twice the inelastic contribution (summed over all dipole configurations):

$$\sigma_{\gamma^*A} = 2 \int \frac{dz d^2p_\perp}{2(2\pi)^3} |\psi_{\gamma^*}^{q\bar{q}}(z, p_\perp, Q^2)|^2 \pi R_A^2, \quad (3)$$

here z is the fraction of the photon's plus momentum, $q_+ = q_0 + q_3$, carried by the quark, $z = p_+/q_+$, and p_\perp is the modulus of its transverse momentum in the plane perpendicular to photon's direction (the anti-quark has $(1-z)$ and $-p_\perp$). In eq. (3) we implicitly assume that the interaction is diagonal in z and p_\perp and black for each configuration. It is convenient to define

$$M^2 = (p_\perp^2 + m_q^2)/(z(1-z)) \quad (4)$$

for the mass-squared of the $q\bar{q}$ system and angle θ between the direction of the momentum of the quark in the centre of mass frame and the photon direction (the transverse plane is defined to be perpendicular to this). Neglecting m_q^2 compared to M^2, Q^2 gives

$$\sin \theta = 2p_\perp/\sqrt{M^2}, z = (1 + \cos \theta)/2. \quad (5)$$

Hence for the nuclear structure functions we get:

$$\begin{aligned} F_T^A(x, Q^2) &= \int_0^{M_{\max}^2} dM^2 \frac{2\pi R_A^2}{12\pi^3} \frac{Q^2 M^2 \rho(M^2)}{(M^2 + Q^2)^2} \\ &\times \int_{-1}^1 d\cos \theta \frac{3}{8} (1 + \cos^2 \theta), \end{aligned} \quad (6)$$

$$F_L^A(x, Q^2) = \int_0^{M_{\max}^2} dM^2 \frac{2\pi R_A^2}{12\pi^3} \frac{Q^4 \rho(M^2)}{(M^2 + Q^2)^2} \times \int_{-1}^1 d\cos\theta \frac{3}{4} \sin^2\theta, \quad (7)$$

$$\rho(M^2) = \sigma^{e^+e^- \rightarrow \text{hadrons}} / \sigma^{e^+e^- \rightarrow \mu^+\mu^-}. \quad (8)$$

Integration over θ gives the transverse and longitudinal structure functions in the BBL as an integral over produced masses. This expression coincides with the one originally obtained in [1].

The upper cutoff $M_{\max}^2 \leq W^2/(m_N R_A)$ comes from the suppression introduced by the square of the nuclear form factor in the rescattering amplitude. The inclusion of $\rho(M^2)$ corrects the dipole formula, for which eq. (3) holds, for higher order corrections (in α_s) contributing at a given M^2 .

Similarly we obtain for the ratio of the deeply virtual Compton scattering and elastic amplitudes:

$$\frac{M(\gamma_T^* A \rightarrow \gamma A)}{M(\gamma_T^* A \rightarrow \gamma_T^* A)} = \frac{\int_0^{M_{\max}^2} \frac{dM^2 \rho(M^2)}{(M^2 + Q^2)^2}}{\int_0^{M_{\max}^2} \frac{dM^2 \rho(M^2) M^2}{(M^2 + Q^2)^2}}. \quad (9)$$

The use of Gribov's orthogonality argument (to neglect non-diagonal transitions) justifies the removal of the integral over the masses in the above expressions and since diffraction is 50% of the total in the BBL we immediately find for the spectrum of diffractive masses:

$$\frac{dF_T^{D(3)}(x, Q^2, M^2)}{d\cos\theta dM^2} + \frac{\epsilon dF_L^{D(3)}(x, Q^2, M^2)}{d\cos\theta dM^2} = \frac{\pi R_A^2}{12\pi^3} \frac{Q^2 \rho(M^2)}{(M^2 + Q^2)^2} \left(\frac{3}{8} M^2 (1 + \cos^2\theta) + \frac{3}{4} \epsilon Q^2 \sin^2\theta \right), \quad (10)$$

where ϵ is the photon polarization. Thus, the spectrum of hadrons in the centre of mass of the diffractively produced system should be the same as in e^+e^- annihilation. Hence, the dominant diffractively-produced final state will have two jets with a distribution over the centre of mass emission angle proportional to $1 + \cos^2\theta$ for the transverse case, and $\sin^2\theta$ for the longitudinal case. The diffractive cross section, integrated over θ , is obtained

from eq. (10) by removing the $\frac{3}{8}(1 + \cos^2 \theta)$, $\frac{3}{4}\sin^2 \theta$ factors. In this form the answer does not depend on the dominance of $q\bar{q}$ final states. It follows from eq. (10) that in the BBL diffractive production of high p_\perp jets is $\propto M^2$ (while in the LTA it is $\propto \ln Q^2$) and hence is enhanced: $\langle p_\perp^2(jet) \rangle_T = 3 M^2/20$, $\langle p_\perp^2(jet) \rangle_L = M^2/5$.

The relative rate and distribution of jet variables for three jet events (originating from $q\bar{q}g$ configurations) will also be the same as in e^+e^- annihilation and hence is given by the standard expressions for the process $e^+e^- \rightarrow q\bar{q}g$. In addition, in the BBL, the production of jets is flavor democratic (weighted by quark charges but unrelated to the quark content of the target).

An important advantage of the diffractive BBL signal is that these features of the diffractive final state should hold for $M^2 \leq Q_{\text{BBL}}^2$ even for $Q^2 \geq Q_{\text{BBL}}^2 \gg \Lambda_{\text{QCD}}^2$ because configurations with transverse momenta $\leq Q_{\text{BBL}}/2$ are perturbative but still interact in the black regime (and correspond to transverse size fluctuations for which the interaction is already black).

Another interesting feature of the BBL is the spectrum of leading hadrons in the photon fragmentation region. It is essentially given by eq. (10). Since the distributions in z (or equivalently θ) do not depend on M , the jet distribution in z is given by

$$\frac{d(\sigma_T + \epsilon\sigma_L)}{dz} \propto \frac{M^2}{Q^2} \frac{1 + (2z - 1)^2}{8} + \epsilon(z - z^2). \quad (11)$$

Exclusive vector meson production in the BBL corresponds, in a sense, to a resurrection of the original vector meson dominance model [10] without off-diagonal transitions. The amplitude for the vector meson-nucleus interaction is proportion to $2\pi R_A^2$ (since each configuration in the virtual photon interacts with the same BBL cross section). This is markedly different from the requirements [11] for matching the generalised vector dominance model (see e.g. [12]) with QCD in the scaling limit, where the non-diagonal matrix elements are large and lead to strong cancellations. We can factorize out the universal black interaction cross section for the dipole interaction from the overlap integral between wave functions of virtual photon and vector meson to obtain for the dominant electroproduction of vector

mesons:

$$\begin{aligned} \frac{d\sigma^{\gamma_T^*+A \rightarrow V+A}}{dt} &= \frac{M_V^2}{Q^2} \frac{d\sigma^{\gamma_L^*+A \rightarrow V+A}}{dt} = \\ &= \frac{(2\pi R_A^2)^2}{16\pi} \frac{3\Gamma_V M_V^3}{\alpha(M_V^2 + Q^2)^2} \frac{4 \left| J_1(\sqrt{-t} R_A) \right|^2}{-t R_A^2}, \end{aligned} \quad (12)$$

where Γ_V is the electronic decay width $V \rightarrow e^+e^-$, α is the fine-structure constant, and $J_1(x)$ is the Bessel function. Thus the parameter-free prediction is that in the BBL at large Q^2 vector meson production cross sections have a $1/Q^2$ behaviour, in stark contrast to an asymptotic behaviour of $1/Q^6$ predicted in pQCD [13].

In the LTA, the factorization theorem is valid and leads to a universal (i.e. target-independent) spectrum of leading particles for scattering off partons of the same flavor. Fundamentally, this can be explained by the fact that, in the Breit frame, the fast parton which is hit by the photon carries practically all of its light-cone momentum ($z \rightarrow 1$). As a result of QCD evolution, this parton acquires virtuality $\sim Q^2$, and a rather large transverse momentum, k_t (which is still $\ll Q^2$). So, in pQCD, quarks and gluons emitted in the process of QCD evolution and in the fragmentation of heavily virtual partons together still carry all the photon momentum. In contrast, in the BBL, configurations in which partons carry all of the photon's momentum form only part of the cross section of leading hadron production. Another piece comes from inelastic collisions of configurations where all partons carry appreciable momentum fractions and large relative transverse momenta (these configurations are rather similar to the case of diffractive scattering (see e.g. eqs. (10, 11))).

The inclusive spectrum of leading hadrons can be estimated as due to the independent fragmentation of quark and antiquark of virtualities $\geq Q^2$, with z and p_\perp distributions given by eqs. (10, 11), (cf. diffractive production of jets discussed above). The independence of fragmentation is justified because large transverse momenta of quarks dominate in the photon wave function (cf. eqs. (4, 5, 6, 7)) and because of the weakness of the final state interaction between q and \bar{q} , since the α_s is small and the rapidity interval is of the order of one. Obviously, this leads to a gross depletion of the leading hadron spectrum as compared

to the LTA situation in which leading hadrons are produced in the fragmentation region of the parton which carries essentially all momentum of the virtual photon. If we neglect gluon emissions in the photon wave function, we find, for instance, for the differential multiplicity of leading hadrons, $dN_{T^*/h}^*/dz$, produced by transverse virtual photons, in the BBL,

$$\frac{dN_{T^*/h}^*}{dz} = 2 \int_z^1 D^{q/h}(z/y, Q^2) \frac{3}{4} (1 + (2y - 1)^2) dy . \quad (13)$$

Here $D^{q/h}(z/y, Q^2)$ is the fragmentation function of quark with flavor q into hadrons. To simplify eq. (13) we used $D^{u/h} = D^{d/h}$ and neglected a rather small difference between $D^{u/h}$ and $D^{s,c/h}$.

Calculations using eq. (13) show a strong suppression of the leading hadron spectrum as compared to the LTA predictions, see Fig.1. Moreover we expect a further softening as Q^2 increases, resulting from increased parton emission in the virtual photon wave function: progressively more configurations contain extra hard gluons, each fragmenting independently in the BBL, further amplifying deviations from the standard LTA predictions.

Another important signature of the BBL is the hardening of p_\perp distributions with decreasing x (at fixed Q^2). Hence, an efficient experimental strategy would be to select leading jets in the current fragmentation region and examine their z and p_\perp -dependence as a function of x . Qualitatively, the effect of broadening of p_\perp -distributions is similar to the increase of the p_\perp -distribution in [15], although final states in DIS were not discussed in this model.

An important advantage of incoherent scattering is the ability to select scattering at central impact parameters (for example via studies of inelastic interactions as a function of the number of nucleons produced in the nucleus fragmentation region). Such a selection allows the effective thickness of the nucleus to be increased, as compared to the inclusive situation, by a factor of ~ 1.5 , and hence allows the BBL to be reached at significantly larger x . An important signature will be a strong depletion of the leading hadron spectrum with centrality in the BBL, and a lack of correlations if the leading twist dominates.

To summarize, we have demonstrated that the study of final states in DIS off nuclei would provide several stringent signals for the onset of the BBL. In contrast to inclusive

observables they will not depend on details of the input parton distributions, and hence will be far more reliable. In the nucleon target case, despite the fact that many photon configurations are far away from the BBL, an examination of these final states observables appears to be a promising way to search for precursors of the BBL. In particular, looking for a strong depletion of hadrons in the current fragmentation region, in combination with the detection of particles in the proton fragmentation region, looks promising.

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FIGURES

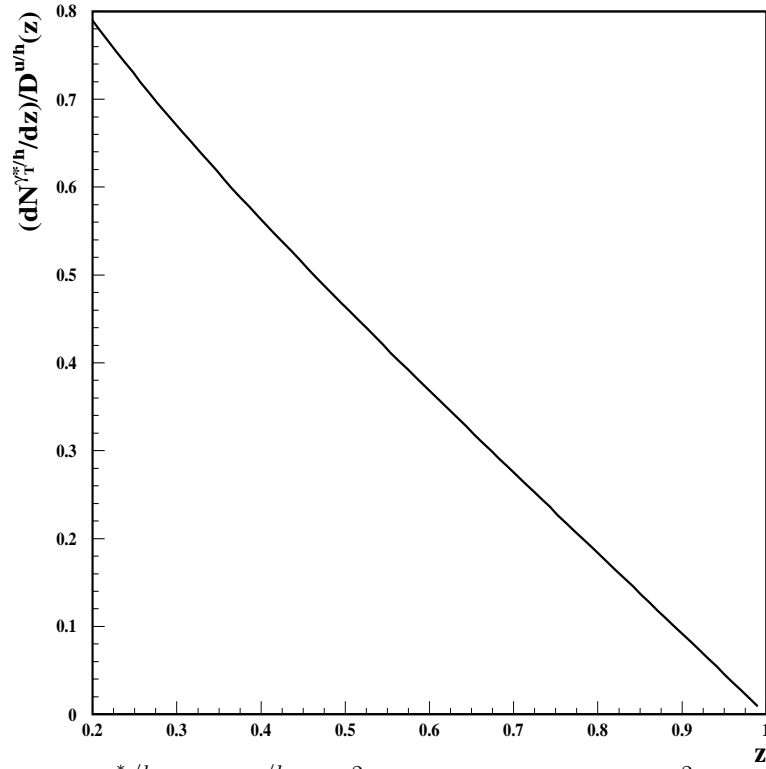


FIG. 1. The ratio $(dN^{\gamma_T^*/h}/dz)/D^{u/h}(z, Q^2)$ as a function of z at $Q^2=2 \text{ GeV}^2$. $D^{u/h}(z, Q^2)$ is from [14].

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